Reputation vs. Litigation

Based on a paper by Dellarocas

Alon Altman

alon@vipe.technion.ac.il

Technion, IIT

Introduction

- Economic activity requires agents to abide by promises.

Introduction

- Economic activity requires agents to abide by promises.
- Cooperation can result from threat of litigation, or by means of published reputation.

Introduction

- Economic activity requires agents to abide by promises.
- Cooperation can result from threat of litigation, or by means of published reputation.
- We discuss a reputation-based mechanism for a model where a *seller* offers a product or service to one of a set of *buyers*.

Introduction

- Economic activity requires agents to abide by promises.
- Cooperation can result from threat of litigation, or by means of published reputation.
- We discuss a reputation-based mechanism for a model where a *seller* offers a product or service to one of a set of *buyers*.
  - The seller promises the good is of high quality.

Introduction

- Economic activity requires agents to abide by promises.
- Cooperation can result from threat of litigation, or by means of published reputation.
- We discuss a reputation-based mechanism for a model where a *seller* offers a product or service to one of a set of *buyers*.
  - The seller promises the good is of high quality.
  - However, the seller may cheat an exert low effort, thus reducing the probablity of generating a high quality good.

The Economic Environment

- Monopolist seller offers unit of good each period.

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.
- Buyer i has valuation w_i for the high quality good and 0 for the low quality good.

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.
- Buyer i has valuation w_i for the high quality good and 0 for the low quality good.
- Discount factor for the seller is δ.

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.
- Buyer i has valuation w_i for the high quality good and 0 for the low quality good.
- Discount factor for the seller is δ.
- Seller may exert low or high effort:

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.
- Buyer i has valuation w_i for the high quality good and 0 for the low quality good.
- Discount factor for the seller is δ.
- Seller may exert low or high effort:
  - **Low effort** — the good is of low quality with probability β.

The Economic Environment

- Monopolist seller offers unit of good each period.
- m i.i.d. buyers each period, each buyer participates in exactly one period.
- Buyer i has valuation w_i for the high quality good and 0 for the low quality good.
- Discount factor for the seller is δ.
- Seller may exert low or high effort:
  - **Low effort** — the good is of low quality with probability β.
  - **High effort** — the seller incurs an additional cost c, the good is of low quality with probability α ($\alpha < \beta$).

Second-price auction

- Assume without loss of generality

$$w_1 \geq w_2 \geq \cdots \geq w_m.$$

- We assume a second-price auction
  - The good goes to the highest bidder (buyer 1).
  - Buyer 1 pays the second-highest bid G of buyer 2.
- The results equally apply to any other reasonable mechanism.

Reputation framework

- Winning buyers acurately experience and report the quality of the good received as *positive* or *negative*.

Reputation framework

- Winning buyers acurately experience and report the quality of the good received as *positive* or *negative*.
- The mechanism publishes the number of negative reports in the past N periods as a number $x \in \{0, 1, \ldots, N\}$.

Reputation framework

- Winning buyers acurately experience and report the quality of the good received as *positive* or *negative*.
- The mechanism publishes the number of negative reports in the past N periods as a number $x \in \{0, 1, \ldots, N\}$.
- Reports older than N periods are discarded.

Reputation framework

- Winning buyers acurately experience and report the quality of the good received as *positive* or *negative*.
- The mechanism publishes the number of negative reports in the past N periods as a number $x \in \{0, 1, \ldots, N\}$.
- Reports older than N periods are discarded.
- Dellarocas has shown that the maximum attianable efficiency for this kind of mechanism is independent of N, so we assume $N = 1$.

Reputation framework

- Winning buyers acurately experience and report the quality of the good received as *positive* or *negative*.
- The mechanism publishes the number of negative reports in the past N periods as a number $x \in \{0, 1, \ldots, N\}$.
- Reports older than N periods are discarded.
- Dellarocas has shown that the maximum attianable efficiency for this kind of mechanism is independent of N, so we assume $N = 1$.
- Therefore, $x \in \{0, 1\}$, where $x = 0$ means good reputation and $x = 1$ means bad reputation.

Reputation stage game

1. Seller offers unit of good, promising high quality.
2. System provides binary rating for seller, based on quality of good received by most recent buyer.
3. Buyers bid thier expected valuations for the good. Winning bidder pays the second-highest bid G.
4. Seller decides whether to exert high effort at cost c, or low effort at cost 0, with corresponding probabilies that the resulting good is of low quality being α and β ($\alpha < \beta$).
5. Buyer receives the good and realizes valuation w_1 for a high quality good or 0 for a low quality good, then accurately reports quality of the good to the system.

Stage game analysis

- Assume a stationary strategy for the seller $\mathbf{s} = [s(0), s(1)]$.

Stage game analysis

- Assume a stationary strategy for the seller $\mathbf{s} = [s(0), s(1)]$.
- As we assume a second-price auction, the buyers bid their expected valuation

$$
\begin{aligned}
G_i(x, \mathbf{s}) &= [s(x) \cdot (1-\alpha) + (1-s(x)) \cdot (1-\beta)]w_i \\
&= [s(x) \cdot (\beta - \alpha) + (1 - \beta)]w_i.
\end{aligned}
$$

Stage game analysis

- Assume a stationary strategy for the seller $\mathbf{s} = [s(0), s(1)]$.
- As we assume a second-price auction, the buyers bid their expected valuation

$$\begin{aligned} G_i(x, \mathbf{s}) &= [s(x) \cdot (1-\alpha) + (1-s(x)) \cdot (1-\beta)]w_i \\ &= [s(x) \cdot (\beta - \alpha) + (1 - \beta)]w_i. \end{aligned}$$

- Thus, the expected auction revenue is

$$G(x, \mathbf{s}) = [s(x) \cdot (\beta - \alpha) + (1 - \beta)]w_2.$$

Expected surplus

- For the winning bidder:

$$V_b(x, \mathbf{s}) = [s(x) \cdot (\beta - \alpha) + (1 - \beta)](w_1 - w_2).$$

- For the seller:

$$\begin{aligned} V_s(x, \mathbf{s}) &= G(x, \mathbf{s}) - s(x) \cdot c \\ &= [s(x) \cdot (\beta - \alpha) + (1 - \beta)]w_2 - s(x) \cdot c. \end{aligned}$$

Seller's future payoff

$$U(x,\mathbf{s}) = s(x)\cdot U_{coop}(x,\mathbf{s}) + (1-s(x))\cdot U_{cheat}(x,\mathbf{s})$$

- For cooperation:

$$U_{\text{coop}}(x,\mathbf{s}) = -c+\delta\left[\begin{array}{ll} & (1-\alpha)(G(0,\mathbf{s})+U(0,\mathbf{s})) \\ + & \alpha(G(1,\mathbf{s})+U(1,\mathbf{s})) \end{array}\right]$$

- . . . and for cheating:

$$U_{\text{cheat}}(x,\mathbf{s}) = \delta\left[\begin{array}{ll} & (1-\beta)(G(0,\mathbf{s})+U(0,\mathbf{s})) \\ + & \beta(G(1,\mathbf{s})+U(1,\mathbf{s})) \end{array}\right]$$

Seller's incentive compatibility

- Note that

$$
\begin{aligned}
U_{\text{coop}}(0, \mathbf{s}) &= U_{\text{coop}}(1, \mathbf{s}) = U_{\text{coop}}(\mathbf{s}) \\
U_{\text{cheat}}(0, \mathbf{s}) &= U_{\text{cheat}}(1, \mathbf{s}) = U_{\text{cheat}}(\mathbf{s})
\end{aligned}
$$

- An equilibirum strategy s for the seller must satisfy

$$
\begin{aligned}
U_{\text{coop}}(\mathbf{s}) > U_{\text{cheat}}(\mathbf{s}) &\Rightarrow s(0) = s(1) = 1 \\
U_{\text{coop}}(\mathbf{s}) = U_{\text{cheat}}(\mathbf{s}) &\Rightarrow 0 \leq s(0), s(1) \leq 1 \\
U_{\text{coop}}(\mathbf{s}) < U_{\text{cheat}}(\mathbf{s}) &\Rightarrow s(0) = s(1) = 0
\end{aligned}
$$

Skip Proofs

Stategy 1: Always cooperate

- Assume $U_{\text{coop}} > U_{\text{cheat}}$. Then $s(0) = s(1) = 1$.

Stategy 1: Always cooperate

- Assume $U_{\text{coop}} > U_{\text{cheat}}$. Then $s(0) = s(1) = 1$.
- So, $G(0, \mathbf{s}) = G(1, \mathbf{s}) = G(\mathbf{s})$ and $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$

Stategy 1: Always cooperate

- Assume $U_{\text{coop}} > U_{\text{cheat}}$. Then $s(0) = s(1) = 1$.
- So, $G(0, \mathbf{s}) = G(1, \mathbf{s}) = G(\mathbf{s})$ and $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$
- But,

$$
\begin{aligned}
U_{\text{coop}} - U_{\text{cheat}} &= -c + \delta[(1-\alpha)(G(0,\mathbf{s}) + U(0,\mathbf{s}))] + \alpha(G(1,\mathbf{s}) + U(1,\mathbf{s}) \\
&\quad -\delta[(1-\beta)(G(0,\mathbf{s}) + U(0,\mathbf{s})) + \beta(G(1,\mathbf{s}) + U(1,\mathbf{s}))] \\
&= -c + \delta(\beta - \alpha)[G(\mathbf{s}) + U(\mathbf{s}) - (G(\mathbf{s}) + U(\mathbf{s}))] \\
&= -c < 0
\end{aligned}
$$

Stategy 1: Always cooperate

- Assume $U_{\text{coop}} > U_{\text{cheat}}$. Then $s(0) = s(1) = 1$.
- So, $G(0, \mathrm{s}) = G(1, \mathrm{s}) = G(\mathrm{s})$ and $U(0, \mathrm{s}) = U(1, \mathrm{s}) = U(\mathrm{s})$
- But,

$$\begin{aligned} U_{\text{coop}} - U_{\text{cheat}} &= -c + \delta[(1-\alpha)(G(0,\mathrm{s}) + U(0,\mathrm{s}))] + \alpha(G(1,\mathrm{s}) + U(1,\mathrm{s}) \\ &\quad -\delta[(1-\beta)(G(0,\mathrm{s}) + U(0,\mathrm{s})) + \beta(G(1,\mathrm{s}) + U(1,\mathrm{s}))] \\ &= -c + \delta(\beta - \alpha)[G(\mathrm{s}) + U(\mathrm{s}) - (G(\mathrm{s}) + U(\mathrm{s}))] \\ &= -c < 0 \end{aligned}$$

- Therefore, this is not a viable strategy.

Strategy 2: Always cheat

- Assume $U_{\text{coop}} < U_{\text{cheat}}$. Then, $s(0) = s(1) = 0$.

Strategy 2: Always cheat

- Assume $U_{\text{coop}} < U_{\text{cheat}}$. Then, $s(0) = s(1) = 0$.
- As before, $G(0, \mathbf{s}) = G(1, \mathbf{s}) = G(\mathbf{s})$ and $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$

Strategy 2: Always cheat

- Assume $U_{\text{coop}} < U_{\text{cheat}}$. Then, $s(0) = s(1) = 0$.
- As before, $G(0, \mathrm{s}) = G(1, \mathrm{s}) = G(\mathrm{s})$ and $U(0, \mathrm{s}) = U(1, \mathrm{s}) = U(\mathrm{s})$
- and also,

$$U_{\text{cheat}} - U_{\text{coop}} \;=\; c > 0$$

Strategy 2: Always cheat

- Assume $U_{\text{coop}} < U_{\text{cheat}}$. Then, $s(0) = s(1) = 0$.
- As before, $G(0, \mathbf{s}) = G(1, \mathbf{s}) = G(\mathbf{s})$ and $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$
- and also,

$$U_{\text{cheat}} - U_{\text{coop}} \;=\; c > 0$$

- Therefore, this is always an equilibrium strategy

Mixed strategy

- Assume $U_{\text{coop}} = U_{\text{cheat}}$.

Mixed strategy

- Assume $U_{\text{coop}} = U_{\text{cheat}}$.
- Note that still $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$.

Mixed strategy

- Assume $U_{\text{coop}} = U_{\text{cheat}}$.
- Note that still $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$.
- So now

$$-c + \delta[(1-\alpha)(G(0,\mathbf{s}) + U(\mathbf{s}))]+\alpha(G(1,\mathbf{s}) + U(\mathbf{s}))$$
$$= -\delta[(1-\beta)(G(0,\mathbf{s}) + U(\mathbf{s}))+\beta(G(1,\mathbf{s}) + U(\mathbf{s}))]$$

Mixed strategy

- Assume $U_{\text{coop}} = U_{\text{cheat}}$.
- Note that still $U(0, \mathbf{s}) = U(1, \mathbf{s}) = U(\mathbf{s})$.
- So now

$$-c + \delta[(1-\alpha)(G(0,\mathbf{s}) + U(\mathbf{s}))] + \alpha(G(1,\mathbf{s}) + U(\mathbf{s}))$$
$$= -\delta[(1-\beta)(G(0,\mathbf{s}) + U(\mathbf{s})) + \beta(G(1,\mathbf{s}) + U(\mathbf{s}))]$$

- Thus,

$$\begin{aligned} c &= \delta(\beta-\alpha)[G(0,\mathbf{s}) - G(1,\mathbf{s})] \\ &= \delta(\beta-\alpha)(s(0) - s(1))(\beta-\alpha)w_2 \\ s(0) - s(1) &= \frac{c}{w_2\delta(\beta-\alpha)^2} > 0 \end{aligned}$$

Mixed Strategy (cont.)

- But as $0 \leq s(0), s(1) \leq 1$, $s(0) - s(1) \leq 1$.

Mixed Strategy (cont.)

- But as $0 \leq s(0), s(1) \leq 1$, $s(0) - s(1) \leq 1$.
- Therefore,

$$\frac{c}{w_2 \delta(\beta - \alpha)^2} \leq 1$$

or

$$\frac{w_2}{c} \geq \frac{1}{\delta(\beta - \alpha)^2}$$

is a pre-requisite for this mixed strategy equilibrium.

The equilibrium strategies

- Never cooperate (s $= [0, 0]$) is an equilibrium stategy.
- If $\frac{w_2}{c} \geq \frac{1}{\delta(\beta-\alpha)^2}$, then the seller's strategy

$$[s(0), s(0) - \frac{c}{w_2\delta(\beta-\alpha)^2}]$$

is an equilibrium strategy for all

$$\frac{c}{w_2\delta(\beta-\alpha)^2} \leq s(0) \leq 1.$$

- These are all equilibrium startegies for the seller in this game.

Optimal equilibrium

The optimal equilibrium strategy $\mathbf{s}^*$ is defined as the strategy that maximizes the seller's lifetime payoff

$$W(\mathbf{s}) = G(x_0, \mathbf{s}) + U(\mathbf{s}),$$

where x_0 is the initial reputation profile, as well as the winning buyer's expected single period surplus.

Optimal equilibrium

- The optimal strategy for the seller is:
  - If $\frac{w_2}{c} < \frac{1}{\delta(\beta-\alpha)^2}$, $\mathbf{s}^* = [0, 0]$.
    The stage-game auction revenue is then
    $$G(x, \mathbf{s}) = (1 - \beta)w_2.$$
  - If $\frac{w_2}{c} \geq \frac{1}{\delta(\beta-\alpha)^2}$, $\mathbf{s}^* = [1, 1 - \frac{c}{w_2\delta(\beta-\alpha)^2}]$.
    The stage-game auction revenue is then
    $$G(x, \mathbf{s}) = (1 - \alpha)w_2 - x\frac{c}{\delta(\beta - \alpha)}.$$

Total Surplus

- The single stage total surplus $V(x, \mathbf{s}) = V_b(x, \mathbf{s}) + V_s(x, \mathbf{s})$ is equal to

$$V(x, \mathbf{s})[s(x)(\beta - \alpha) + (1 - \beta)]w_1 - s(x)c$$

- The *average* single stage total surplus is

$$V(\mathbf{s}) = p_0(\mathbf{s})V(0, \mathbf{s}) + p_1(\mathbf{s})V(1, \mathbf{s})$$

where $p_0(\mathbf{s})$, $p_1(\mathbf{s})$ are stationary probabilities that a seller who follows strategy s will find himself in states $x = 0$, $x = 1$ repectively.

Total Surplus

- If $\frac{w_2}{c} < \frac{1}{\delta(\beta-\alpha)^2}$, the average total surplus is

$$V(\mathbf{s}) = (1-\beta)w_1$$

- If $\frac{w_2}{c} \geq \frac{1}{\delta(\beta-\alpha)^2}$, the average total surplus is

$$V(\mathbf{s}^*) = (1-\alpha)w_1 - c - \left(\frac{\alpha c}{\beta - \alpha}\right)\frac{w_1(\beta-\alpha)-c}{\delta w_2(\beta-\alpha)-c}$$

Litigation

Litigation game

1. Seller offers unit of good, promising high quality.
2. Buyers bid thier expected valuations for the good. Winning bidder pays the second-highest bid G.
3. Seller decides whether to exert high effort at cost c, or low effort at cost 0, with corresponding probabilies that the resulting good is of low quality being α and β ($\alpha < \beta$).
4. Buyer receives the good and realizes valuation w_1 for a high quality good or 0 for a low quality good.

Litigation game (cont'd)

5. Buyer decides whether or not to sue the seller. If the buyer chooses not to sue, the game ends.
6. If the buyer sues, both buyer and seller incur a cost L. The court finds for the buyer with probability a if the good was of high quality and with probability b ($b > a$) if the good was of low quality.
7. If the court finds for the buyer, the seller has to pay the buyer damages D.

Analysis

- If $L > bD$, the buyer will never sue and the seller will always exert low effort.
- If $L < aD$, the buyer will always sue and the seller will exert high effort iff

$$D > \frac{c}{(\beta - \alpha)(b - a)}$$

- If $aD < L < bD$, then the buyer will sue iff a good of low quality is received and the seller will exert high effort iff

$$c < (\beta - \alpha)(L + bD)$$

Total surplus

- If $c < (\beta - \alpha)(L + bD)$ and $L < \frac{(\beta-\alpha)w_1 - c}{2\alpha}$, then social surplus is maximized by setting $\frac{L}{b} < D < \frac{L}{a}$. Then, the seller always exerts high effort and the buyers sues iff she receives a low quality good, and the average total surplus is

$$V = [(1 - \alpha)w_1 - c] - 2\alpha L$$

- Otherwise, total surplus is maximized by setting damages $D < \frac{L}{b}$. Then, the seller always exerts low effort, but the buyer never sues and the average total surplus is

$$V = (1 - \beta)w_1$$