Proof that $b(v) = \frac{n-1}{n+1-k}v$ is an equilibrium in k-price auction:

- Reverse the bid function to reveal the value:

$$v = \frac{n+1-k}{n-1}b$$

- Probability distribution over bids (for the other traders):

$$\Pr[b < x] = \frac{n+1-k}{n-1}x$$

- Probability of winning with a bid of b:

$$P_{win} = \left(\frac{n+1-k}{n-1}b\right)^{n-1}$$

- Profit from a winning bid of b:

$$v - \frac{n-k+1}{n}b$$

- Best response is to maximize your profit:

$$b = \underset{b}{\operatorname{argmax}} \left[\left(\frac{n+1-k}{n-1}b\right)^{n-1}\left[v - \frac{n-k+1}{n}b\right]\right]$$

- Differenitate to find the maximum

$$\begin{aligned}
b' &= (n-1)\left(\frac{n+1-k}{n-1}b\right)^{n-2}\frac{n+1-k}{n-1}\left[v - \frac{n-k+1}{n}b\right] + \\
&- \frac{n-k+1}{n}\left(\frac{n+1-k}{n-1}b\right)^{n-1} = 0 \\
\frac{n-k+1}{n}\left(\frac{n+1-k}{n-1}b\right) &= (n-1)\frac{n+1-k}{n-1}\left[v - \frac{n-k+1}{n}b\right] \\
\frac{n+1-k}{n-1}b &= (n-1)\left[v - \frac{n-k+1}{n}b\right] \\
\frac{n+1-k}{n-1}b &= (n-1)v - \frac{(n-1)(n-k+1)}{n}b \\
(n+1-k)b &= (n-1)v \\
b &= \frac{n-1}{n+1-k}v
\end{aligned}$$